Truth Tables

and

Equivalent Statement Forms

The NEGATION OF statement p is a statement $\sim p$ (called "NOT p") which in all cases has the opposite truth value as p has.

p	$\sim p$
T	F
F	T

Some rules for assigning truth values:

p	q	$p \wedge q$ ("p AND q")	$p \vee q$ ("p OR q")	$\sim p$
T	T	T	T	F
T	F	F	T	F
F	T	F	T	T
F	F	F	F	T

Exercise:

Determine the truth table of the statement form: $(p \wedge \sim q) \vee s$

THE TRUTH TABLE FOR $(p \wedge \sim q) \vee s$

p	q	s	$\sim q$	$p \wedge \sim q$	$(p \wedge \sim q) \vee s$
T	T	T	F	F	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	F	F	T
F	T	F	F	F	F
F	F	T	T	F	T
F	F	F	T	F	F

EXAMPLE: Either this Monday is a holiday and we don't have to go to school or I will stay home anyway.

Question: UNDER WHAT CONDITIONS WILL IT be a lie to say this?

Use p = "Monday is a holiday."
q = "We have to go to school."
s = "I stay home."

Definition: Two statement forms (with the same variables) are logically equivalent ($\equiv$) if they have the same truth table (and so they have the same truth value in all possible cases).

EXAMPLE:

DE MORGAN'S LAWS:

① $\sim(p \wedge q) \equiv (\sim p) \vee (\sim q)$

② $\sim(p \vee q) \equiv (\sim p) \wedge (\sim q)$

Proof of ②:

p	q	$(p \vee q)$	$\sim(p \vee q)$	$(\sim p)$	$\wedge$	$(\sim q)$
T	T	T	F	F	F	F
T	F	T	F	F	F	T
F	T	T	F	T	F	F
F	F	F	T	T	T	T

Because these are the same, $\sim(p \vee q) \equiv (\sim p) \wedge (\sim q)$